ECONOMETRICS

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Estimator and Its Properties

(Chapter 6-8 : Newbold)

Estimator and Its Properties

- **Estimator (Def.)** An "estimator" or "point estimate" is a statistic that is used to infer the value of an unknown parameter in a statistical model.
  - The parameter being estimated is sometimes called the *estimand*. If the parameter is denoted by θ then the estimator is typically written by adding a "hat" over the symbol, $\hat{\theta}$.
  - The attractiveness of different estimators can be judged by looking at their properties, such as unbiasedness, efficiency, consistency, etc..
  - The construction and comparison of estimators are the subjects of estimation theory.

Estimator and Its Properties

* Estimator's properties can be divided into two categories:

- Small Sample Properties
  - Unbiasedness
  - Efficiency
  - Sufficiency

- Large Sample Properties
  - Consistency

Estimator and Its Properties

SMALL SAMPLE PROPERTIES

1. **Unbiasedness**: An estimator is said to be unbiased if its expectation equals the value of the population parameter. That is, if:

$$\boxed{E(\hat{\theta}) = \theta, \text{ then } \hat{\theta} \text{ is an unbiased estimator of } \theta}$$

Could you show this graphically? – try and get a positive!

Examples: $E(\bar{X}) = \mu$ *(Proof 1)*

$E(s^2) = \sigma^2$ *(Proof 2)*

Remark: Intuitively, an unbiased estimator is 'right on target'

Estimator and Its Properties

SMALL SAMPLE PROPERTIES

1. **Unbiasedness** ⟶ **BIAS**

Let $\hat{\theta}$ be an estimator of θ.

The Bias is defined as the difference between its mean and θ.

$$\boxed{\text{Bias}(\hat{\theta}) = E(\hat{\theta}) - \theta}$$

Remark: the bias of an unbiased estimator is equal to 0.

Estimator and Its Properties

SMALL SAMPLE PROPERTIES

2. **Efficiency**: An estimator is said to be efficient if in the class of unbiased estimators it has minimum variance. See this in more detail:

- Suppose there are several unbiased estimators of θ.
- The Most Efficient Estimator (or Minimum Variance Unbiased Estimator) is the unbiased estimator with the smaller variance.
  - **Example**: Let $\hat{\theta}_1$ and $\hat{\theta}_2$ be the two unbiased estimators of θ, based on the same number of observations. Then

$$\boxed{\hat{\theta}_1 \text{ is said to be more efficient than } \hat{\theta}_2 \text{ if } Var(\hat{\theta}_1) < Var(\hat{\theta}_2)}$$

Estimator and Its Properties

SMALL SAMPLE PROPERTIES

2. <u>**Efficiency**</u> → **Relative Efficiency**

$$\boxed{\text{Relative efficiency} = \frac{\mathrm{Var}(\hat{\theta}_2)}{\mathrm{Var}(\hat{\theta}_1)}}$$

Estimator and Its Properties

SMALL SAMPLE PROPERTIES

2. **Efficiency**:

And, *what happens when there is conflict between efficiency and unbiasedness???* (i.e. $\hat{\theta}_1$ is unbiased, $\hat{\theta}_2$ is biased but $Var(\hat{\theta}_1) > Var(\hat{\theta}_2)$)

- In these cases, we need to compute the Mean Square Error (MSE):

$$MSE(\hat{\theta}) = Var(\hat{\theta}) + \text{bias}(\hat{\theta})^2$$

$\hat{\theta}_2$ is preferred to $\hat{\theta}_1$ if $MSE(\hat{\theta}_2) < MSE(\hat{\theta}_1)$

- **Remark:** for an unbiased estimator, the MSE is simply the variance of the estimator

Estimator and Its Properties

SMALL SAMPLE PROPERTIES

3. **Sufficiency**: We say that an estimator is sufficient if it uses all the sample information.

- The median, because it considers only rank, is not sufficient.
- The sample mean considers each member of the sample as well as its size, so is a sufficient statistic. Or, given the sample mean, the distribution of no other statistic can contribute more information about the population mean.

Estimator and Its Properties

LARGE SAMPLE PROPERTIES

4. **Consistency**:

- Let $\hat{\theta}$ be an estimator of θ.
- $\hat{\theta}$ is a consistent estimator of θ if the difference between the expected value of $\hat{\theta}$ and θ decreases as the sample size increases.
- Consistent estimators are desirable when unbiased estimators can not be obtained.

Appendix

Appendix

SUMS OF RANDOM VARIABLES

Let X_1, X_2,…, X_K K random variables with means μ_1, μ_2,…, μ_K and variances ${\sigma_1}^2$, ${\sigma_2}^2$, …, ${\sigma_K}^2$. The following properties are satisfied:

1. The mean of its sum is the sum of their means, that is,

$$E(X_1 + X_2 + \ldots + X_K) = \mu_1 + \mu_2 + \ldots + \mu_K$$

2. If the covariance between each pair of these random variables is 0, then the variance of the sum is the sum of their variances, that is,

$$\mathrm{Var}(X_1 + X_2 + \ldots + X_K) = {\sigma_1}^2 + {\sigma_2}^2 + \ldots + {\sigma_K}^2$$

However, if the covariance between pair of random variables is not 0,

$$\mathrm{Var}(X_1 + X_2 + \ldots + X_K) = {\sigma_1}^2 + {\sigma_2}^2 + \ldots + {\sigma_K}^2 + 2\sum_{i=1}^{K-1}\sum_{j=i+1}^{K} Cov(X_i, X_j)$$

Appendix

DIFFERENCES BETWEEN A PAIR OF RANDOM VARIABLES

Let X and Y a pair of random variables with means μ_X and μ_Y and variances ${\sigma_X}^2$, ${\sigma_Y}^2$. The following properties are satisfied:

1. The mean of its difference is the difference of their means, that is,

$$E(X - Y) = \mu_X - \mu_Y$$

2. If the covariance between X and Y is 0, then the variance of its difference is

$$\mathrm{Var}(X - Y) = {\sigma_X}^2 + {\sigma_Y}^2$$

3. If the covariance between X and Y is not 0, then the variance of its difference is

$$\mathrm{Var}(X - Y) = {\sigma_X}^2 + {\sigma_Y}^2 - 2\mathrm{Cov}(X, Y)$$

Appendix

LINEAR COMBINATION OF RANDOM VARIABLES

The linear combination of two random variables X and Y is: $Z = aX + bY$ where a and b are constants.

1. The mean value of Z is

$$\mu_Z = E(Z) = E(aX + bY) = a\mu_X + b\mu_Y$$

2. The variance of Z is

$$\sigma_Z{}^2 = a^2\sigma_X{}^2 + b^2\sigma_Y{}^2 + 2ab\mathrm{Cov}(X, Y)$$

Or using the correlation,

$$\sigma_Z{}^2 = a^2\sigma_X{}^2 + b^2\sigma_Y{}^2 + 2ab\mathrm{Corr}(X, Y)\sigma_X\sigma_Y$$